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ADVANCED RELIABILITY ANALYSIS FOR ENGINEERING STRUCTURES THROUGH MACHINE LEARNING METHOD

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Abstract. *In structural reliability analysis and design practice, the most probable point (MPP)-based methods are still widely adopted. However, unwanted computational efforts and inaccuracy are still concerns when dealing with engineering structures. Within the framework of the first-order reliability method (FORM), an advanced MPP capturing method assisted by a supervised machine learning technique, namely the Extended Support Vector Regression (X-SVR), is introduced. In order to elevate the robustness of the X-SVR, a newly developed generalized kernel function is presented as a supplementary selection for kernel mapping. Through the X-SVR technique, the surrogate model can be established by alternatively depicting the underpinned relationship between the system uncertainties and the concerned structural response through a mathematical function with the explicit expression. By formulating the optimization problem on the established surrogate model, both gradient-based and metaheuristics algorithms can be implemented to capture the MPP effectively and efficiently. Finally, a numerical investigation is thoroughly investigated to demonstrate the applicability and computational efficiency of the proposed scheme.*

1 INTRODUCTION

Structural failure is the most undesirable event that could happen, as it can result in extremely severe consequences. Even if the structure is designed to be safe, the uncertainty is still a non-neglectable risk of the structure which results in a failure probability to some extent [1]. In engineering practice, the limit state design principle is often used to ensure the integrity of structures during their intended life cycle. Adequate safety against different failure modes is often given by calibrating the load and resistance factors [2]. However, traditional semi-probabilistic approaches can lead to over-conservative designs. Thus, an effective approach to quantify the risk of failure is important using structural reliability analysis approaches.

In structural reliability analysis, the probability of failure P_f can be found as the solution of a multi-integration problem which is expressed as [3],

$$P_f = P\left(g\left(\mathbf{x}\right) < 0\right) = \int \cdots \int_{g(\mathbf{x})<0} f_{\mathbf{X}}\left(\mathbf{x}\right) \mathrm{d}\mathbf{x} \tag{1}$$

where $g\left(\mathbf{x}\right)$ denotes the limit state function; $f_{\mathbf{X}}(\mathbf{x})$ denotes the joint probability density function (PDF) of the random variables vector $\mathbf{x}$ with n-dimensions.

It is extremely difficult to solve such integration of the joint PDF analytically, apart from the dimensionality involved in engineering structures. In practice, the Monte Carlo Simulation (MCS) runs a large number of limit state functions to estimate the probability of failure and is widely used due to its simplicity of execution. As the number of required simulations is often excessive for MCS, the MCS becomes computationally infeasible for time-consuming implicit limit state functions. To address this shortcoming, various variation reduction methods are developed [4]. These methods include importance sampling (IS), line sampling (LS), directional sampling (DS) and subset sampling (SS). Regardless of the improvement in computational efficiency, these simulation-based methods still rely on sampling and evaluating the original time-consuming limit state functions. Furthermore, information on failure regions is often a prerequisite of these methods. High nonlinearity and multi-dimensionality can cause additional cumbersome effort due to this reason. The First Order Reliability Method (FORM) is extensively used to mitigate such challenges [5]. FORM approximates the limit state function using the first-order Taylor expansion and searches the so-called most probable point (MPP) to evaluate the reliability index. The probability of failure can be computed from the level of the reliability index.

The problem of solving the multi-integration of joint PDF is converted into an equality-constrained optimization problem with the help of FORM. Various optimization algorithms are developed which are categorized into two types, gradient-based algorithms and metaheuristic algorithms [6]. However, both methods require the repeated evaluation of the original limit state function. Unwanted computational cost is still a challenge yet to be solved by FORM. To alleviate this challenge, this paper introduces the use of the Extended Support Vector Regression (X-SVR) assisted advanced MPP searching method. The X-SVR is utilized to establish the surrogate model, inspired by its previous achievements in uncertainty quantifications [7-11]. The great performance in regression by X-SVR is supported by the underlying mathematical theory as the hyperplane searching is formulated into a convex, Quadratic Programming (QP) optimization problem.

The paper is organized as follows. The FORM is reviewed in Section 2. The proposed advanced reliability analysis through the X-SVR is introduced in Section 3. In Section 4, a numerical model of a truss dome is investigated to demonstrate the applicability of the proposed method for an implicit limit state function. Section 5 concludes the paper.

2. FIRST-ORDER RELIABILITY METHODS (FORM)

FORM is widely used in structural reliability analysis and designs due to its simplicity and reduced number of function evaluations. The sensitivity of failure probability due to system variables can also be obtained. In the FORM, the reliability index is evaluated using the MPP from the equality-constrained optimization problem as

$$\begin{aligned} &\text{find } \mathbf{u}^* \\ &\beta = \min(\mathbf{u}^{\mathrm{T}} \cdot \mathbf{u})^{1/2} \\ &s.t. \;\; G(\mathbf{u}) = 0 \end{aligned} \tag{2}$$

where $\mathbf{u}^*$ is defined as the optimal solution of the MPP and $\mathbf{u}$ denotes the vector of random variables in standard normal space Ψ. However, Rosenblatt transformation is required to map the limit state function from the original design space Ω to standard normal space Ψ if the random variables do not follow a standard normal distribution. This transformation is visualized in Figure 1.

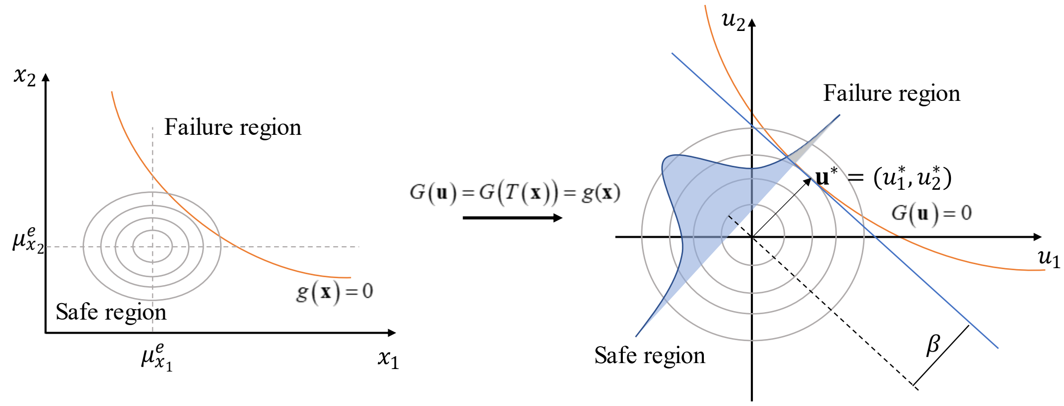


Figure 1. The transformation from Ω -space to Ψ -space for capturing the MPP.

Even though the FORM is widely applied in practice, it still has four major limitations: (1) The FORM relies on the first Tylor expansion of the limit state function. As a result, the accuracy of FORM is dependent on the linearity of the limit state functions. (2) The transformation between design space and standard normal space may introduce errors. (3) High computational costs for structures involving complex geometry, high dimensionality, and sophisticated physical problems. (4) The global optimal cannot be guaranteed.

3. ADVANCED RELIABILITY ANALYSIS THROUGH EXTENDED SUPPORT VECTOR REGRESSION (X-SVR)

In this section, the framework of the advanced structural reliability analysis through the machine learning technique is illustrated and explained. As this work intends to reduce the computational burden during reliability analysis, the supervised learning technique, so-called X-SVR is adopted herein to obtain the surrogate model to the original time-consuming limit state function. Through the establishment of the surrogate model, the original implicit limit state function can be replaced with an explicit mathematical expression that can be analyzed in an efficient manner. In order to further increase the robustness of the existing X-SVR, the Bernoulli polynomial is integrated with the Gaussian kernel to form the Generalized Bernoulli Kernel (GBK). The detailed algorithm is listed in Table 1.

Table 1. The algorithm of the proposed method

Algorithm 1. Advanced reliability analysis for engineering structures through X-SVR

Stage I: Surrogate model construction

1.1. Generate the training inputs;
1.2. Compute the corresponding training outputs following the physical relationship;
1.3. Construct the surrogate model through the X-SVR technique;
1.4. If the surrogate model is converged:

1.4.1. Export the surrogate model $\hat{g}(\mathbf{x})$ and set $i=0$;

1.4.2. Initialize the searching point as $\mathbf{x}_0 = [\mu_1, \mu_2, ..., \mu_n]$;

1.5. Else
1.5.1. Increase additional training input datasets;
1.5.2. Go back to Step 1.2;
End If.

Stage II: Surrogate model-based MPP capturing scheme

2.1. Transform the established surrogate model $\hat{g}(\mathbf{x}_i)$ from Ω -space to $\hat{G}(\mathbf{u})$ Ψ -space at the current searching point;

2.2. Capture the next searching point $\mathbf{u}_{i+1}$ using gradient-based or metaheuristic solvers;

2.3. Transform $\mathbf{u}_{i+1}$ from Ψ -space to $\mathbf{x}_{i+1}$ in Ω -space;

2.4. If $\left\|\mathbf{u}_{i+1}-\mathbf{u}_i\right\|<1\times10^{-6}$

2.4.1 Terminate the iteration;

2.5. Else

2.5.1 $i=i+1$;

2.5.2. Go back to step 2.1.

End If

2.6. Return the MPP as $\mathbf{x}^*=\mathbf{x}_i$.

Stage III: Reliability index evaluation

3.1. Evaluate the reliability index as $\beta=\left\|\mathbf{u}^*\right\|$.

The framework of the proposed method consists of three stages which are illustrated in Table 1. The three stages include I. Surrogate model construction, II. Surrogate model based MPP capturing scheme, and III. Reliability index evaluation.

Stage I contains the procedures for surrogate model construction. In order to obtain high-quality samples, the design of experiment (DoE) scheme uses the uniform design method to cover the design space for all system inputs. The training inputs and outputs are generated using Latin Hypercube Sampling (LHS) method following this DoE scheme to achieve thorough coverage. Afterwards, the X-SVR technique can be used to establish the surrogate model.

Stage II contains the surrogate model-assisted MPP capturing scheme, in which the established surrogate model in the design space Ω is transformed into standard normal space Ω using Rosenblatt transformation $\hat{G}(\mathbf{u})=\hat{G}(T(\mathbf{x}))=\hat{g}(\mathbf{x})\approx g(\mathbf{x})$. Due to the nature of the equality-constrained optimization problem, both gradient-based and metaheuristic optimization algorithms can be applied to conduct the MPP searching tasks based on the established surrogate model. For gradient-based algorithms, the initial searching point is selected as the mean value for each variable as $\mathbf{x}_0=[\mu_1,\mu_2,...,\mu_n]$. Then, the response $\hat{G}(\mathbf{u}_i)$ and gradient vectors $\nabla\hat{G}(\mathbf{u}_i)$ of the surrogate model can be evaluated at each iteration. For both types of algorithms, the next searching point can be expressed as $\mathbf{u}_{i+1}\hat{g}(\mathbf{x})$. The convergence criterion can be described as $\left\|\mathbf{u}_{i+1}-\mathbf{u}_i\right\|<1\times10^{-6}$. Once the convergence criterion is met, the MPP can be found as $\mathbf{x}^*=\mathbf{x}_i$.

In Stage III, the reliability index can be evaluated based on the last search point $\mathbf{u}_{i+1}$ as $\beta=\left\|\mathbf{u}^*\right\|$.

Compared to the traditional MPP searching FORM algorithms, the proposed **Algorithm 1.** has several advantages that are discussed as follows,

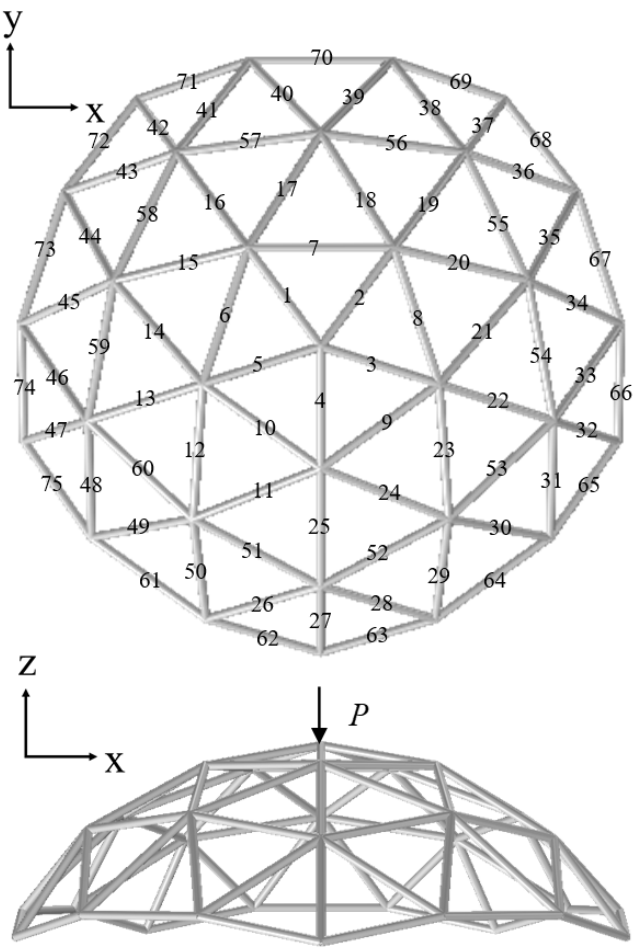
(1) For the proposed algorithm, the computational costs are mainly taken by the generation of the training set. Therefore, the computational burden can be drastically alleviated by reducing the number of function evaluations of the original time-consuming limit state function.

(2) When the limit state condition is changed or multiple limit states are concerned, the same surrogate model can still be applicable. On the other hand, the traditional FORM needs to be undertaken repeatedly.

(3) The constructed surrogate model is free from any numerical noise or data imperfection that prevents the traditional FORM from converging or affecting its converging speed. Therefore, more stable and predictable performance can be achieved.

4. CASE STUDY

To demonstrate the applicability of the proposed X-SVR-assisted structural reliability framework, a geodesic truss dome is investigated. The numerical model and geometry of the truss are depicted in Figure 2.



Node	x (m)	y (m)	z (m)
1	0.000	0.000	0.000
2	-4.004	0.000	0.709
3	-1.237	-3.808	0.709
4	3.240	-2.354	0.709
5	3.240	2.354	0.709
6	-1.237	3.808	0.709
7	-8.009	0.000	3.183
8	-5.729	-4.162	2.395
9	-2.475	-7.617	2.883
10	2.188	-6.735	2.395
11	6.479	-4.708	3.183
12	7.082	0.000	2.395
13	6.479	4.708	3.183
14	2.188	6.735	2.395
15	-2.475	7.617	3.183
16	-5.729	4.162	2.395
17	-10.434	0.000	6.448
18	-9.246	-3.808	5.658
19	-6.479	-7.617	5.658
20	-3.224	-9.923	6.448
21	0.765	-9.971	5.658
22	5.252	-8.516	5.658
23	8.441	-6.133	6.448
24	9.719	-2.354	5.658
25	9.719	2.354	5.658
26	8.441	6.133	6.448
27	5.242	8.516	5.658
28	0.765	9.971	5.658
29	-3.224	9.923	6.448
30	-6.479	7.617	5.658
31	-9.246	3.808	5.658

Figure 2. The numerical model of the geodesic truss dome.

In Figure 2, the truss dome structure is illustrated which consists of 75 bar members and 31 nodes. All members are numbered with nodal coordinates provided. The 15 bottom nodes are simply supported, and a concentrated force is acting on top of the truss. The serviceability limit state is selected, and the maximum global deflection allowed at any node is 0.035m. The limit state function can be expressed as,

$$g\left(\mathbf{x}\right) = 0.035 - \Delta_z(\mathbf{x}) \tag{3}$$

where Δ_Z represents the maximum deflection of the truss dome, which is dependent on the geometry, material and loading parameters of the structure, including (A_i , for $i = 1, 2, ..., 75$) which denotes the cross-section areas for all bars; E denotes Young's modulus of the material; and P denotes the load on the top. These terms are regarded as random variables and their statistical information can be found in Table 2.

Table 2. Statistical information of random variables for truss dome [6]

Variables	$A_1 - A_5$ (m^2)	$A_6 - A_{10}$ (m^2)	$A_{11} - A_{25}$ (m^2)	$A_{26} - A_{50}$ (m^2)	$A_{51} - A_{60}$ (m^2)	$A_{61} - A_{75}$ (m^2)	E (GPa)	P (kN)
Mean	0.0025	0.002	0.001	0.0012	0.0022	0.0015	70	80
CoV	0.15	0.12	0.08	0.1	0.1	0.1	0.05	0.15
Distribution type	Normal	Normal	Normal	Normal	Normal	Normal	Lognormal	Gumbel

The convergence study shows that 500 training data is sufficient. To validate the performance of the surrogate model, the *RMSE* and the R^2 have been evaluated as 9.247×10^{-5} and 0.9999, respectively. The subsequent reliability analysis and MPP searching are conducted based on this model. The results for FORM and the proposed method are tabulated in Table 3. The MCS with 5×10^5 is calculated as the reference.

Table 3. Estimated MPP and reliability index of the truss dome.

Method	MCS	HL-RF	FSL	X-SVR-SQP	X-SVR-FSL
A^*_{1-5} (m^2)	-	0.002139778	0.002139777	0.002122753	0.0025
A^*_{6-10} (m^2)	-	0.00191819	0.00191819	0.001916521	0.001895614
A^*_{11-25} (m^2)	-	0.000994155	0.000994155	0.000994583	0.000993793
A^*_{26-50} (m^2)	-	0.001198594	0.001198594	0.001197966	0.001198064
A^*_{51-60} (m^2)	-	0.002192858	0.002192858	0.002192517	0.00218881
A^*_{61-75} (m^2)	-	0.0015	0.0015	0.001500316	0.001497335
E^* (GPa)	-	68.2534	68.2534	68.2409	68.1417
P^* (kN)	-	108.8484	108.8484	108.2317	113.8488
Eval	5×10^5	2475	2925	500	500
β	2.1502	2.2566	2.2566	2.2540	2.2751
RE (%)	-	4.95	4.95	4.83	5.81

It is shown in Table 3, the 5×10^5 times of MCS sets the reference as the $\beta_{\text{MCS}} = 2.1502$. The FORM algorithms HL-RF and FSL yield identical MPP. However, the FSL takes a greater number of evaluations of the original limit state function and HL-RF due to the finite length of the searching step. It is notable that the proposed advanced MPP searching method provides a more accurate reliability index through the X-SVR-SQP algorithm compared to the traditional FORM. More importantly, both X-SVR-SQP and X-SVR-FSL only evaluated the original limit state function 500 times. As for more detailed computational time, the MCS consumes 2972.6 minutes. The HL-RF and FSL consume 3514.5 s and 4135.5 s, respectively. However, based on the surrogate model based upon 500 samples, the proposed method only consumes 1.01 s and 0.44 s for X-SVR-SQP and X-SVR-FSL, respectively. It can be found from the results that the proposed method drastically reduces the computational time and yields an accurate result for the investigated case. This method can be further expanded in engineering practice.

5. CONCLUSION

In summary, this work aims to propose an advanced most probable point (MPP) searching method through a machine learning technique, namely the Extended Support Vector Regression (X-SVR). The X-SVR is implemented to establish the surrogate model for the original computational exhaustive limit state functions. A new kernel function is also embedded in the training process. The three main stages of the method are (1) Surrogate model constructions; (2) FORM-based MPP capturing and (3) result processing. The MPP is searched based on the proposed method. A truss dome is investigated to demonstrate the applicability and efficiency of the proposed method. The performance of the prediction of the established surrogate model using the X-SVR technique is validated. Furthermore, the reliability index and corresponding MPP can be captured efficiently and effectively.

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