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NUMERICAL STUDY ON THE BEHAVIOUR OF A UNIFORMLY STRESSED METALLIC TOROIDAL PRESSURE VESSEL OVERWRAPPED WITH CARBON FIBER REINFORCED POLYMERS

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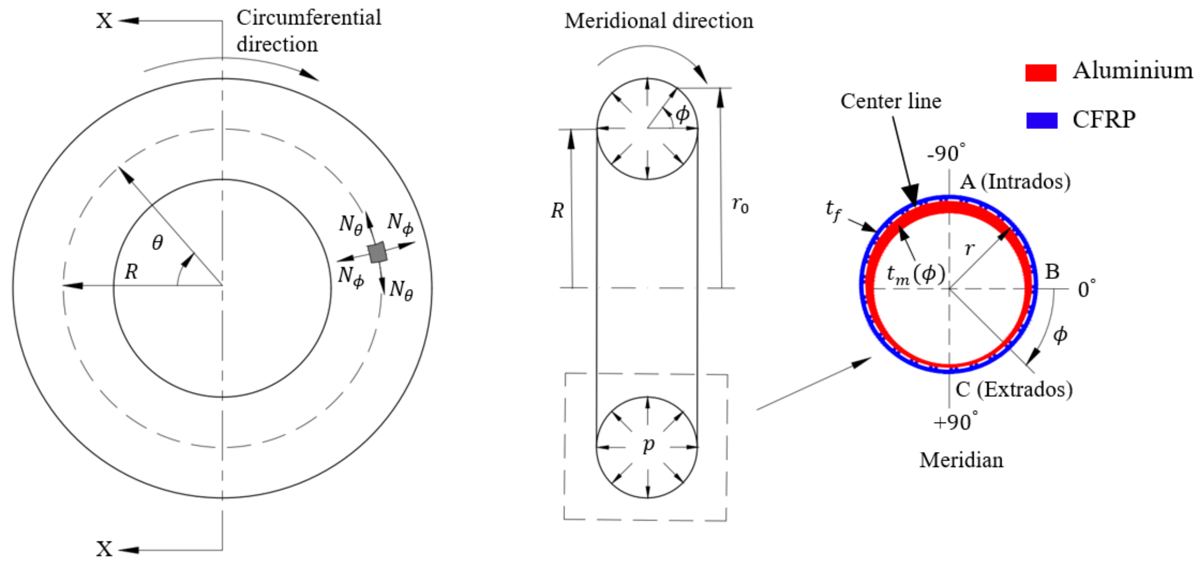
Abstract. *Toroidal pressure vessels (TPVs) are an alternative to cylindrical and spherical pressure vessels for fluid storage. However, uniformly thick TPVs have non-uniform stresses along the meridian, with higher stresses near the intrados and lower stresses near the extrados. This non-uniform stress distribution results in an ineffective utilization of the material's strength, and the TPV often fails along the intrados before the material yields along the extrados. To resolve this issue, TPVs can have varying wall thickness along the meridian, which also reduces the weight of the vessel. Wrapping fiber-reinforced polymers, such as carbon fiber-reinforced polymer (CFRP), over the metallic liner can further enhance the TPV's capacity with minimal weight increase. This study aims to investigate the effect of wrapping CFRP over a uniformly stressed aluminum TPV with a circular cross-section. A finite element analysis using the ABAQUS has been conducted to understand the behavior of the uniformly stressed CFRP over-wrapped TPV.*

1 INTRODUCTION

Toroidal pressure vessels (TPVs) are an alternative solution to the traditional available cylindrical and spherical pressure vessels. The TPVs are majorly used in space limited applications, where there is a restriction on height and length availability. However, the structural performance of the toroid is better than the conventional pressure vessels, due to its uniform thickness distribution and absence of end domes/caps [1]. The toroidal shape is at least as efficient as a cylinder [2]. The TPVs have many advantages over cylindrical pressure vessels, weight saving (absence of end domes), space saving, and fixed position of center of mass (due to geometry). The geometry of the TPV is shown in Figure 1. Where r is the cross-sectional radius, R is the toroidal radius, ϕ is the meridional angle, and θ is the circumferential angle.

A TPV with uniform thickness subjected to uniform internal pressure results in a non-uniform stress distribution along the meridian. The von Mises stresses are higher near the intrados and lower near the extrados. The non-uniformity in the stresses is due to the convex/ concave curvature of the shell surface and the difference in the surface area in the inner and outer region of the TPV [3]. This non-uniform stress distribution results in un-utilization of the material near the extrados (i.e., the vessel fails near intrados even before it yields near the extrados in few cases, where R/r is less). Three ways can overcome this problem a) increasing R/r ratio, b) by composite overwrapping [4], and c) providing non-uniform thickness along the meridian [5]. In addition, it also helps in improving the storage/structural efficiency of the TPV. The structural performance efficiency is calculated by pressure times volume divided by the weight of the vessel.



a) Geometry b) Cross-section showing thickness distribution

Figure 1: Geometry of the FRP over-wrapped uniformly stressed TPV

Vu [5], proposed a simpler solution to find the optimal cross-section of the toroid based on the non-uniform thickness distribution along the meridian calculated using linear membrane solution. Paleti et al. [4], developed a theoretical solution for metal-FRP TPV based on modified linear membrane solution. Li and Cook [2], proposed a linear membrane solution for filament wound TPV. The proposed solution is based on winding dry fibers on the liner without any matrix in the meridional direction, results in neglecting the effect of matrix in the circumferential direction. Tielking et al [6], proposed a modified linear membrane solution based on energy based approach for pressurized TPV. From the previous research works, it is concluded that there are linear membrane solutions available for metallic and filament wound TPV. However, a membrane solution for metal-FRP TPV that considers the effect of matrix is not available. In addition, there are no studies on the effect of FRP on the uniformly stressed TPV.

In the present work, a linear membrane solution for CFRP over-wrapped uniformly stressed aluminium TPV is proposed and verification of the proposed solution (PS) is carried out by finite element analysis (FEA). The effect of number of layers of CFRP on uniformly stresses aluminium TPV is also studied. In addition, results of CFRP over-wrapped uniformly stressed TPV are compared with CFRP over-wrapped constant thick aluminium TPV.

2 LINEAR MEMBRANE SOLUTION

The linear membrane theory (LMT) is often used to analyse the pressure vessels under internal pressure [23,26]. The linear membrane theory assumes that the bending stiffness is negligible, and the applied loads are resisted only by in-plane forces generated in the shell wall. The LMT results in the exact prediction of the membrane stresses, if the bending stresses developed are negligible. For thin-walled structures, where radius to thickness ration is very high the bending stresses have least effect on the results. Therefore, in the present formulations it is assumed that the thickness of the shell is very small compared to meridional radius. In addition, a perfect bond between base metal and FRP is assumed to simplify the problem. The stress resultants can be calculated by writing the equilibrium equations of the shell element [7]. Therefore, the meridional and circumferential forces in the toroidal pressure vessel from membrane theory of shells is given by

$$\text{Meridional membrane force, } N_\phi = \frac{pr}{2}\frac{2R+rsin\,\phi}{R+rsin\,\phi} \tag{1}$$

$$\text{Circumferential membrane force, } N_\theta = \frac{pr}{2} \tag{2}$$

where p is the internal pressure. The membrane stress can be found on dividing the membrane forces with the shell thickness.

$$\text{von Mises stress, } \sigma_v = \sqrt{\sigma_\phi^2 + \sigma_\theta^2 - \sigma_\phi\sigma_\theta}$$

$$= \frac{pr}{2t_l(R+rsin\,\phi)}\sqrt{3R^2 + 3Rrsin\,\phi + r^2\sin^2\phi} \tag{3}$$

where σ_ϕ and σ_θ are the meridional and circumferential stresses, respectively and t_l is the thickness of the base metal.

To produce a uniformly stressed TPV, the wall thickness of the TPV should vary along the meridian. The thickness distribution that produces uniform stress throughout the TPV from Eq. (3) is given by

$$t_l(\phi) = \frac{pr}{2\sigma_v(R+rsin\,\phi)}\sqrt{3R^2 + 3Rrsin\,\phi + r^2\sin^2\phi} \tag{4}$$

The von Mises stress in the TPV is constant, if the wall thickness is provided as per the Eq (4). The relationship between membrane forces, stresses in the base metal and the FRP, is given by

$$N_\phi = t_l\,\sigma_{\phi l} + t_f\,\sigma_{\phi f} \tag{5}$$

$$N_\theta = t_l\,\sigma_{\theta l} + t_f\,\sigma_{\theta f} \tag{6}$$

where, $\sigma_{\phi l}$ and $\sigma_{\phi f}$ are the merdional stresses in the base metal and FRP; $\sigma_{\theta l}$ and $\sigma_{\theta f}$ are the circumferential stresses in the base metal and FRP and t_f is the thickness of the FRP lamina.

The stress-strain relationship for base metal and FRP are given by

$$\epsilon_{\phi l} = \frac{1}{E}\left(\sigma_{\phi l} - \nu\sigma_{\theta l}\right) \tag{7}$$

$$\epsilon_{\theta l} = \frac{1}{E}\left(\sigma_{\theta l} - \nu\sigma_{\phi l}\right) \tag{8}$$

$$\epsilon_{\phi f} = \left(S_{\phi\phi}\,\sigma_{\phi f} + S_{\phi\theta}\,\sigma_{\theta f}\right) \tag{9}$$

$$\epsilon_{\theta f} = \left(S_{\theta\theta}\,\sigma_{\theta f} + S_{\phi\theta}\,\sigma_{\phi f}\right) \tag{10}$$

where, $S_{\phi\phi} = \frac{1}{E_1}$ $\quad S_{\theta\theta} = \frac{1}{E_2}$ $\quad S_{\phi\theta} = -\frac{\nu_{12}}{E_1}$

where $\epsilon_{\phi l}$ and $\epsilon_{\phi f}$ are the meridional strains in the base metal and FRP; $\epsilon_{\theta l}$ and $\epsilon_{\theta f}$ are the circumferential strains in the base metal and FRP.

It is assumed that there is perfect bond between the base metal and FRP. Therefore, for perfect bond between the base metal and FRP,

$$\epsilon_{\phi l} = \epsilon_{\phi f} \text{ and } \epsilon_{\theta l} = \epsilon_{\theta f} \tag{11}$$

Therefore, from above Eqs. (7-11) we have

$$\frac{1}{E}\left(\sigma_{\phi l}-\nu\sigma_{\theta l}\right)-\left(S_{\phi\phi}\,\sigma_{\phi f}+S_{\phi\theta}\,\sigma_{\theta f}\right)=0 \tag{12}$$

$$\frac{1}{E}\left(\sigma_{\theta l}-\nu\sigma_{\phi l}\right)-\left(S_{\theta\theta}\,\sigma_{\theta f}+S_{\phi\theta}\,\sigma_{\phi f}\right)=0 \tag{13}$$

By solving equations (5, 6, 12, and 13), the membrane stresses in the base metal ($\sigma_{\phi l}$, $\sigma_{\theta l}$) and FRP ($\sigma_{\phi f}$, $\sigma_{\theta f}$) can be obtained and the strains can be found by using Eqs. (7 and 8).

3 FINITE ELEMENT MODELLING

A three dimensional finite element analysis has been carried out on the CFRP overwrapped uniformly stresses aluminium TPV in ABAQUS [8]. Two toroidal geometries are modelled for base metal and FRP, respectively. The thickness of the base metal shell surface is provided as per Eq. (4). The FE model is discretized with membrane elements (M3D4R) to have a consistent comparison with the linear membrane solution. A perfect bond between base metal and FRP is assigned using tie constraint and a linear static analysis has been carried out in the present study. The material properties used for aluminium and CFRP are shown in Table 1. The FE model of the TPV is shown in Figure 2.

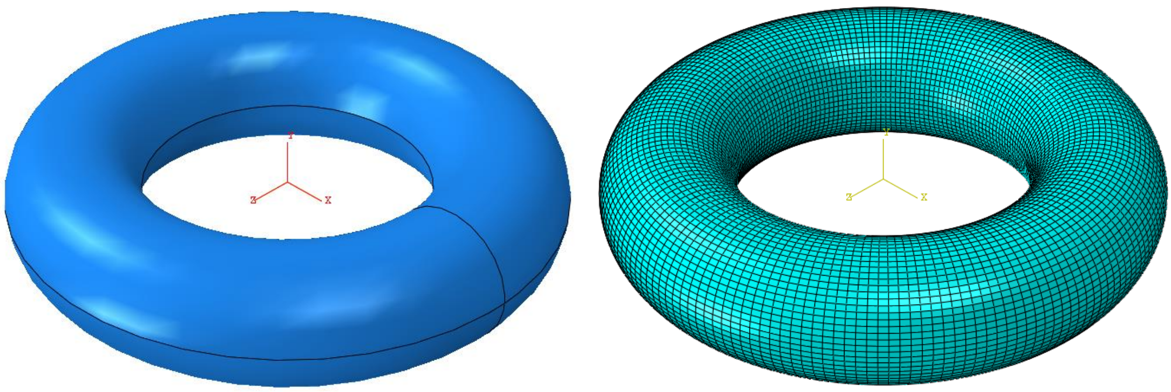


Figure 2: FE model

4 NUMERICAL EXAMPLE

A TPV with toroidal radius R=99.5mm, meridional radius r =31.5mm, for the benchmark model the thickness of the base metal t_m =4mm, and the thickness of CFRP t_f =0.2, 1, and 2mm corresponds to 1, 5, and 10 layers of CFRP is considered. The base metal considered in the present study is aluminium and the overwrap is of CFRP. The properties of aluminium and CFRP are given in Table 1. The thickness of the uniformly stressed aluminium TPV is calculated using Eq. (4), where t_m is 4mm maximum at the intrados and minimum at extrados as shown in Figure 3. The σ_v in Eq. (4) is taken as yield strength of aluminium. The calculated thickness distribution makes the metallic TPV reach yield stress throughout the vessel at same time. The objective of the present work is to study the effect of wrapping FRP on the uniformly stressed aluminium TPV to improve the structural efficiency by using varying thickness along meridian and wrapping FRP.

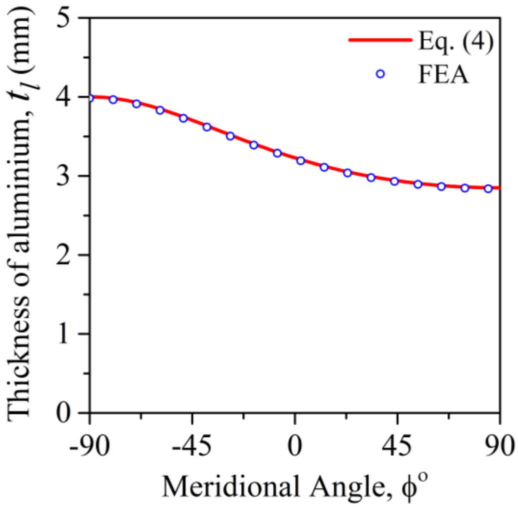


Figure 3: Aluminium thickness distribution along the meridian

Table 1: Material properties of CFRP lamina [9].

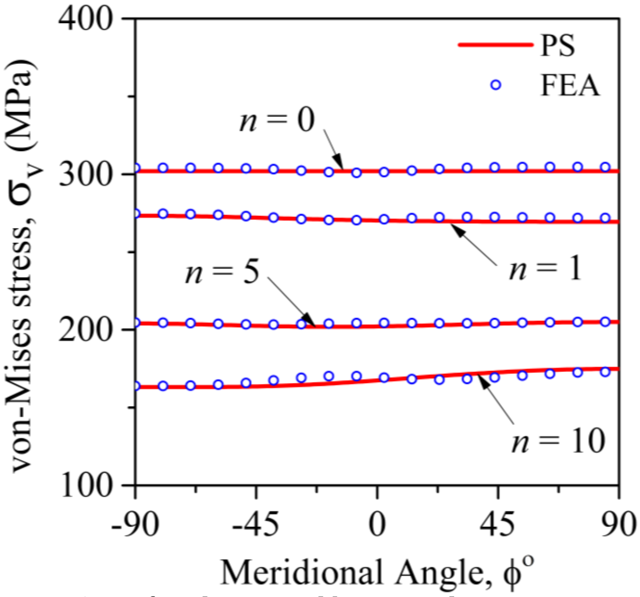
Material property	Value
Modulus of Elasticity of Aluminium, E (GPa)	69.204
Poisson's ratio of Aluminium, ν	0.33
Longitudinal elastic modulus, E_{11} (GPa)	161.74
Transverse elastic modulus, $E_{22} = E_{33}$ (GPa)	9.50
Poisson's ratio, $\nu_{12} = \nu_{13}$	0.33
Poisson's ratio, ν_{23}	0.43
Shear modulus, $G_{12} = G_{13}$ (GPa)	4.57
Shear modulus, G_{23} (GPa)	3.10

5 RESULTS AND DISCUSSION

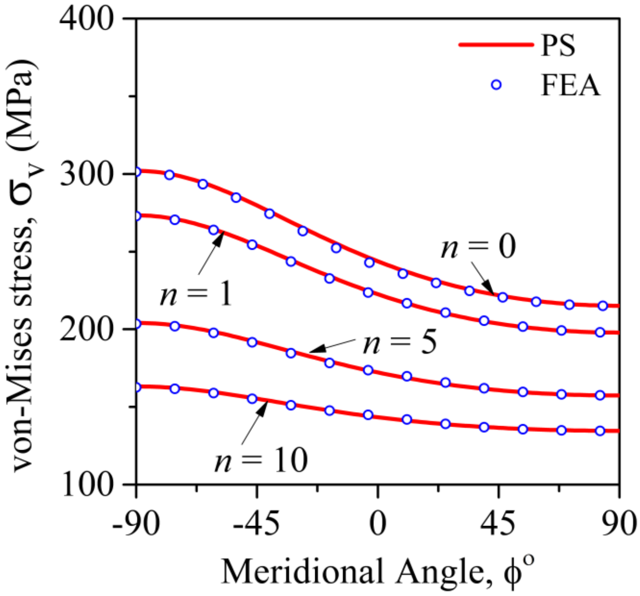
The variation of von-Mises stresses in the aluminum of a CFRP overwrapped uniformly stressed TPV are plotted against meridional angle, shown in Figure 4. The meridional stresses and strains in the aluminum and CFRP from the proposed linear membrane solution are in good agreement with the FEA results. The meridional stresses and strains in the aluminum are almost uniform along the meridian. The meridional stress in the CFRP is non-uniform for n=1, 5, and 10 layers. The non-uniform stresses in CFRP is due to the constant thickness of CFRP along the meridian. Even though only equilibrium condition is considered while deriving LMT the circumferential stresses from LMT are reasonably in good agreement with FEA, unlike constant thick TPV. The maximum percentage error in circumferential strains, < 20% (for n=0) and < 10% for CFRP over-wrapped TPV is observed. The von-Mises stresses in aluminium are uniform throughout the TPV, which represents the condition of a cylinder subjected to internal pressure, shown Figure 4(a). The thickness of the aluminium is provided in such a way that the von-Mises stresses throughout TPV will be uniform and reaches yield stress at a same pressure. It is observed that the stresses in the aluminium decreases with increase in thickness of CFRP. The circumferential strains are increasing with increase in thickness of CFRP due to the poisons effect. From Figure 4(b), it is seen that with the increase in number of layers of CFRP, the von-Mises stresses tend to be uniform along the meridian due to the redistribution of the stresses. The yield pressure for the different cases considered in the present study are given in Table 2.

The yield pressure obtained from LMT and FEA are in good agreement. The percentage of weight difference between the constant thick and uniformly stressed TPV is 14.5%. The same performance is achieved by the uniformly stressed TPV with a reduced weight when compared with constant thick TPV.

With the increase in number of layers of FRP, the stresses near the extrados are greater than the intrados for uniformly stressed FRP over wrapped TPV, shown in Figure 4(a) for n=10. Therefore, with the increase in number of layers of CFRP the aluminium will yield near the extrados instead of intrados. The stresses in CFRP are higher near the intrados and low near the extrados as per the conventional behaviour of TPV. The reason behind this behaviour is uniform thickness of CFRP and redistribution of stresses in the aluminium due to the FRP wrapping. This results in reduced yield capacity of CFRP over-wrapped uniformly stressed TPV with n=10, when compared with constant thick TPV (Table 2).



a) Uniformly stressed base metal-FRP TPV



b) Uniform thick base metal-FRP TPV

Figure 4: von-Mises stress in metal-FRP TPV at p = 35.75 MPa

Table 2: Material properties of CFRP lamina [9].

Number of layers of CFRP	Yield pressure (p_y)			
	Uniformly stressed base metal		Uniform thick base metal	
	FEA	LMT	FEA	LMT
n =0	35.44	35.75	35.55	35.75
n =1	39.28	39.49	39.56	39.49
n =5	52.61	52.66	53.05	52.9
n=10	62.39	61.73	66.38	66.16

6 CONCLUSIONS

The behaviour of uniformly stressed TPV over wrapped with CFRP was studied. A linear membrane solution was presented, and results were then compared with the FEA. A good agreement between results of LMT and FEA has been observed. The structural performance index can be enhanced by using uniformly stressed base metal for metallic and metal-FRP TPV. It has been observed that aluminium yielded near extrados when number of layers of CFRP reached ten. A complete nonlinear analysis of CFRP overwrapped uniformly stressed TPV is recommended to identify the failure location. In addition, a parametric study should be carried out by changing R/r ratio, thickness of the base metal and FRP to draw more generalized conclusions.

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