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ULTIMATE STRENGTH OF TWCFS MEMBERS UNDER ECCENTRIC COMPRESSION – UPPER-BOUND ESTIMATION VIA YIELD LINE ANALYSIS

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Abstract. *The paper includes the results of an analytical-numerical study into the application of YLA to the upper bound estimation of the ultimate strength of short TWCFS lipped channel section members subjected to eccentric compressive load around the minor and major axes. The post-buckling paths are determined with an application of the finite difference method (FDM) and the Newton-Raphson method to obtain equilibrium paths for the bar under eccentric compression. The post-ultimate failure curves are derived using theoretical models of plastic mechanisms, elaborated based on experimental tests and FE simulations. The work method is applied. Results are presented in load-shortening and moment-rotation diagrams, as well as comparative diagrams of the ultimate strength in terms of eccentricity. The ultimate strength is evaluated using upper bound estimation via YLA, FE calculation, experimental tests, and EC3 standard predictions. Some conclusions concerning a potential implementation of upper bound ultimate strength estimation relying on YLA approach are derived. Challenges in successfully implementing this approach and further research in this field are discussed.*

1 INTRODUCTION

Short thin-walled cold-formed steel (TWCFS) members are prematurely prone to local or distortional buckling and do not have a real post-elastic capacity. Failure of such members is initialised by the local-global interactive buckling of plastic-elastic type. Thus, their collapse behaviour, either in compression or bending, is governed by a spatial plastic failure mechanism response. In that case, efficient methods of ultimate strength prediction are necessary, which examine a collapse behaviour, contrary to traditional methods, namely, effective width method or Direct Strength Method [1]. An alternative method occurs to be the Yield Line Analysis (YLA). This method applied to thin-walled steel structures enables an analysis of structural behaviour in the vicinity of the ultimate load and in the post-ultimate stage. It provides a relevant post-ultimate response, e.g. post-ultimate mechanism curve. The intersection of the elastic post-buckling path and the post-ultimate failure mechanism curve provides an upper bound estimation of the member's load carrying capacity (ultimate strength) [2].

On the other extreme, the problem of the load carrying capacity of TWCFS subjected to simple loading systems (pure bending or uniform compression) has been solved with satisfactory accuracy within the theory of thin-walled structures, as well as in design code specifications. EN 1993-1-3 [3] gives accurate predictions for the buckling load and ultimate strength of TWCFS members under concentric axial compression but is less accurate in the examination of eccentrically compressed columns. In that case, the applicability of the YLA approach leading to the upper bound estimation of ultimate strength is worthy of examination.

Bakker [4] classified the analysis of the yield line into two methods: classical and generalised. Classical YLA mainly concerns the load capacity of reinforced concrete structural members. It assumes that only primary bending moments contribute to the plastic strain energy dissipated in the plastic mechanism [1]. Classical YLA goes back to the 20^{th} of the XXth century. Its origins are traced in the works by Ingerslev [5], Rzanicyn [6], and Sawczuk and Jaeger [7]. These works dealt with the analysis of plastic mechanisms (theory of yield line) in the estimation of the load capacity of reinforced concrete plates and slabs. Classical YLA is still used in concrete design specifications, but it cannot be applied directly to the analysis of structural behaviour of thin-walled metallic structures, particularly TWCFS members.

Generalised YLA provides the load-deformation post-ultimate relationship coming from the derivation of the plastic strain energy dissipated in the spatial mechanism due to the second-order displacement fields. This method was initially applied in the 70s of the XXth century to the analysis of post-ultimate behaviour of TWCFS members. Murray [8] reported the first applications in metal structures. The pioneering work was continued by Murray and Khoo [9], Mahendran [10], and Rasmussen and Hancock [11], who analysed the local plastic mechanisms of the column of the channel section and the thin-walled plates under compression. Since then, numerous research reports dealing with the spatial plastic mechanism analysis applied to the evaluation of ultimate and post-ultimate behaviour of TWCFS members have been published. Recently, YLA was applied to the analysis of failure mechanisms and post-ultimate behaviour of corrugated plate structures by Timmers [12] and Casariego et al. [13]. Kobashi et al. [14] applied YLA to the analysis of the deterioration of the strength of relatively long columns with channel section under compression. Kotełko et al. [15] applied YLA to the estimation of the maximum strength of the columns of the perforated lipped channel section subject to compression.

There are relatively few reported results of research into the structural behaviour of TWCFS members under eccentric compression, particularly with respect to major axis bending. A comprehensive state-of-the-art review in this area is given in [16,17]. They indicate that the design rules of existing standards are too conservative and must be improved (in particular, interactive formulae for combined compression and bending). The research was continued by the same authors [18,19]. They presented results of experimental program and theoretical analysis for short members with lipped channel cross-section, subjected to eccentric compression around the minor and major axes, in a large range of eccentricities. Post-ultimate, rigid-plastic curves, characterising a post-ultimate structural behaviour, derived on the basis of plastic mechanism models were compared with results of experiments and FE simulations.

The present paper presents a continuation of the research mentioned above, which attempts to apply the YLA method to ultimate strength estimation of short columns with lipped channel cross-section subjected to eccentric compression around the minor and major axes.

2 SUBJECT AND OBJECTIVES OF THE ANALYSIS

The subject of the present work is to determine the ultimate and post-ultimate behaviour of thin-walled cold-formed steel columns with a lipped channel cross-section under eccentric compression around the minor and major axes, as shown in Figure 1. The dimensions of the columns to investigate were determined in order to classify the members into class 4 according to EN 1993-1-1:2006 [20]. It means that local buckling will occur in such members before the attainment of yield stress in one or more parts of the cross-section.

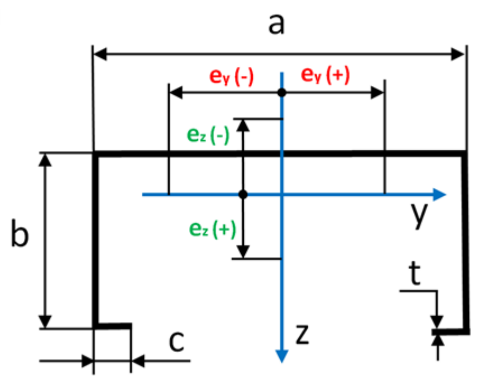

Figure 1: Lipped channel section subject to eccentric compression.

A large research programme was realised, namely theoretical analysis and experimental study on the structural behaviour (including post-ultimate behaviour) of thin-walled cold-formed steel lipped channel section columns subjected to eccentric compression around both the minor and major axes. The results of this research were published in [16-19] The typology of the examined members is shown in Table 1.

Table 1: Dimensions specification

No.	*a* (mm)	*b* (mm)	*c* (mm)	*t* (mm)	*l (mm)*
1	150	60	20	2	450
2	150	60	20	1	450
3	150	47	16	2	450
4	150	47	16	1.5	450
5	150	47	16	1	450
Internal radii r = 1.5 mm (for t = 1 mm) and r = 2.5 mm (for t = 2, 2.5 mm)					

The structural behaviour of the members was theoretically analysed (using FE simulations and the YLA method) and experimentally tested in the wide range of eccentricities from -60 mm to +60 mm (around minor axis) and from 0 to 60 mm (around major axis).

The main objectives of the present study are the following.

- analysis of post-buckling elastic behaviour of examined members in the context of an applicability of post-buckling paths to an upper-bound estimation of the load-carrying capacity of members under investigation;
- upper bound ultimate strength prediction based on post-buckling analysis and YLA approach.

3 METHODOLOGY

As mentioned above, in order to estimate the upper bound load capacity, one has to determine an ordinate of the intersection point of the post-buckling and post-ultimate mechanism failure curve. In the subsequent paragraphs the methodology used to analyse these sections of the member's equilibrium path is described. Equilibrium paths obtained using the methods described in the following were compared with FE simulations and experimental results [16,18,19]

3.1 Pre- and post-buckling analysis

In order to analyse the non-linear behaviour of the thin-walled structure, especially in the post-buckling state, components of the strain tensor, constitutive relations, equations of continuity (between two walls of the member section) and equilibrium equations were determined. It was assumed that a member consisted of thin plates (walls) as shown in Figure 2. For a single thin plate, non-linear strain-displacement relations were defined as follows:

$$\begin{aligned}\varepsilon_x &= u_{,x} + 0.5(v_{,x}^2 + w_{,x}^2) - zw_{,x}\\ \varepsilon_y &= v_{,x} + 0.5w_{,y}^2 - zw_{,y}\\ \gamma_{xy} &= u_{,y} + v_{,x} + w_{,x}w_{,y} - 2zw_{,xy}\end{aligned} \tag{1}$$

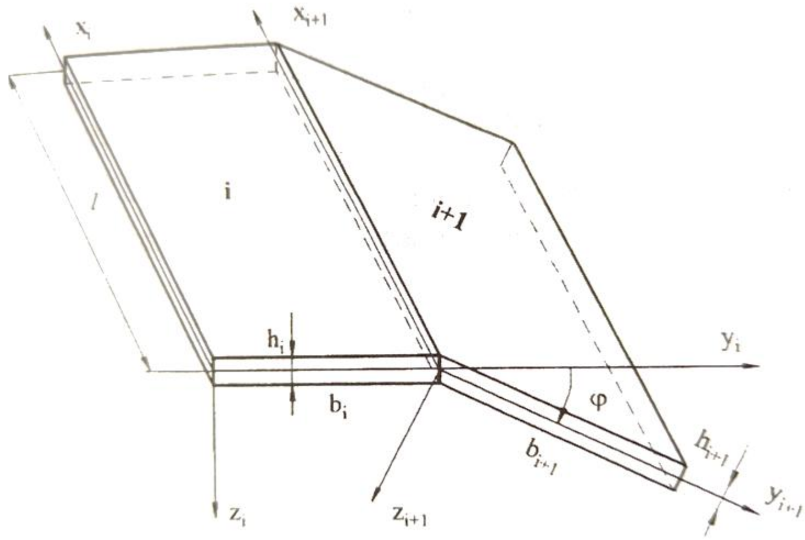

Figure 2: Connection of two walls of thin-walled member.

Constitutive linear relations (generalised Hooke's Law) for isotropic material were assumed. Internal in-plane forces and moments (bending moments and torque) were derived from the classical first-order theory of thin plates [21].

Equilibrium equations were obtained from the variation of the potential energy formulated for a single plate and can be written in the following form:

$$\begin{aligned}&N_{x,x} + N_{xy,x} = 0\\ &N_{y,y} + N_{xy,x} + (N_x v_{,x})_{,x} = 0\\ &M_{x,xx} + 2M_{xy,xy} + M_{y,yy} + (N_x w_{,x})_{,x} + (N_y w_{,y})_{,y} + (N_{xy} w_{,x})_{,y} + (N_{xy} w_{,y})_{,x} = 0\end{aligned} \tag{2}$$

where: [N_x , N_y , N_{xy}] and [M_x , M_y , M_{xy}] are vectors of in-plane sectional forces and moments, respectively.

Equilibrium equations (2) were solved using the Finite Differences Method (FDM) with central difference quotients. The simultaneous non-linear algebraic equation obtained after differentiation of equations (2) was solved using the one-step Newton-Raphson method. This approach takes into account only the global mode of buckling (local modes are omitted).

Boundary conditions for analysed, simply supported structure are:

- $x = 0, L$

$$\sum_{i=1}^{k} \int_0^{b_i} N_{xi}\, dy = F_0$$

$$\sum_{i=1}^{k} \int_0^{b_i} N_{xi} y_{ci}\, dy = F_0 e$$

$$w_i = v_i = 0$$

- $x = 0$

$$u_i = 0$$

- $y = 0, b_5$

$$M_y = N_y = N_{xy} = 0$$

where: L is the length of the member, b_5 is the width of the 5th wall (fold), F_0 is the external compression force, e is the eccentricity, y_{ci} is the coordinate (distance between major/minor axis and node that belongs to i^{th} wall), h is the thickness of the walls, b_i is the width of the i^{th} wall and u, v, w are the displacements (u, v - in-plane displacements in the longitudinal and perpendicular direction, w - deflection).

Pre-buckling loads distribution at plane at x=const. was assumed as:

$$\begin{aligned} N_{x30}(j) &= N_{x20} - j \cdot h_{y3} / b_3 (N_{x20} - N_{x40}) = 0 \\ N_{x10}(j) &= N_{x20} + (b_1 - j \cdot h_{y3}) / b_3 (-N_{x20} + N_{x40}) = 0 \\ N_{x50}(j) &= N_{x40} + j \cdot h_{y1} / b_3 (-N_{x20} + N_{x40}) = 0 \end{aligned} \quad (3)$$

where N_{x40}, N_{x20} are the internal forces for the 4th and 2nd wall respectively, obtained from equilibrium equations written for assumed linear load distribution [N/m], j is the j^{th} node in perpendicular direction.

The discretization of the equilibrium equations enforces the maps of the nodes for every wall, according to Figure 3.

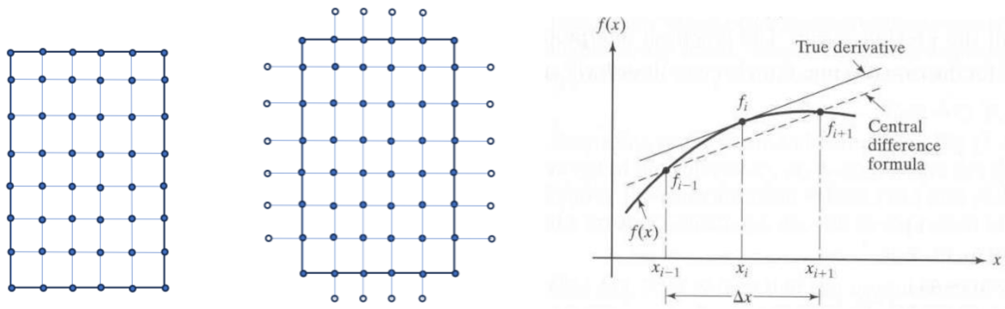

Figure 3: Distribution of nodes for difference quotients obtained for: (a) 'u' and 'v' displacements, (b) for 'w' displacement and (c) definition of central difference quotient [22].

Exemplary definitions of difference quotients are shown below:

$$L_{1,0}(u) = \frac{u_{i+1,j} - u_{i-1,j}}{h_x}$$

$$L_{2,0}(u) = \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h_x^2}$$

$$L_{4,0}(w) = \frac{w_{i+2,j} - 4w_{i+1,j} + 6w_{i,j} - 4w_{i-1,j} + w_{i-2,j}}{h_x^4}$$

where: L is the operator of difference quotient, h_x is the distance between two nodes in 'x' (longitudinal) direction.

The discretization of boundary conditions and equations of continuity leads to the usage of forward and backward quotients.

3.2 Yield-line analysis

The Yield Line Analysis (YLA) assumes that deformations leading to the development of the failure plastic mechanisms are concentrated in zero-width yield lines. The basic assumption is that the plastic mechanism is fully developed, and the plastic zones developed in the walls of thin-walled steel members are concentrated in yield lines, stationary or travelling [23]. At the level of the yield lines, the material is considered to be fully plastic. The material characteristics are assumed to be rigid-perfectly plastic or rigid-plastic with strain hardening.

The theory distinguishes two types of the plastic mechanisms: a so-called “true mechanism” and the so-called “quasi-mechanism”, where the flat parts of the walls are limited by yield lines, but the walls undergo membrane deformation. The plastic mechanism approach is based on two basic methods, namely the *energy method* (work method) and the *equilibrium strip method* [24]. In the present study, only the energy method was applied. Using the energy method, the *Principle of Virtual Velocities* is adopted.

In the case of TWCFS members, the following form of this principle, namely, of the rate of change of the energy dissipated, is usually applied:

$$\dot{\Pi}(\dot{\beta},\chi)=\sum_i \int_{A_i}(N_0\varepsilon_p)dA_i+\sum_i \dot{E}_{bi}(\dot{\beta}_i,\overline{m}_{pi}\chi) \tag{4}$$

where Π is the potential energy of the system.

Equation (4) is a sum of the rate of change of the membrane strain energy in the walls of the global plastic hinge and the bending strain energy dissipated at the yield lines, where N_0 is a vector of membrane forces per unit length. The membrane forces N_0 are determined from the associated flow rule, considering the corresponding yield criterion (e.g. Huber-Mises). A_i is an area of i^{th} plastic zone (tension field) vector, $\dot{\beta}_i$ is an angular velocity vector on the i^{th} yield line and $\overline{m}_{pi}$ is the plastic moment vector on that line.

The first component of equation (4) is taken into account in the case of quasi-mechanisms only, which consist of both yield lines and plastic zones (tension fields). For the true mechanism, only the second component of equation (4) should be considered, where E_{bi} is a bending strain energy dissipated at the i^{th} yield line. After rearranging equation (4) we obtain:

$$\delta W_{ext}=\delta E_b+\delta E_m \tag{5}$$

where δW_{ext} is the variation of work of external forces, δE_b is the variation of the energy of bending plastic deformation, while δE_m is the variation of the energy of membrane plastic deformation. Equation (5) provides a relation of generalised load (e.g. compressive load, bending moment) in terms of general displacement (e.g. shortening, the angle of rotation). The graphical representation of this relation is a failure (post-ultimate) curve.

Alternatively, Equation (5) may be rearranged in the following form:

$$P=\frac{\partial(E_b+E_m)}{\partial\delta} \tag{6}$$

The crucial point in the YLA approach and the biggest challenge is to properly identify yield line patterns and tension fields, in other words, to establish a geometry of the spatial plastic mechanism and a way of the mechanism development. This can be done by experiments and FE simulations. After deriving an initial model of the mechanism, its geometric parameters have to be calibrated in detail on the basis (mainly) of experimental results [17]. In the present paper, the plastic mechanism database is used, elaborated in previous research [16,19]. Theoretical models of plastic mechanisms were derived mainly on the basis of experimental results [17,19]. Failed members were scanned, using the 3D scanning technique, and initially developed theoretical models were calibrated. The procedure is shown in Figure 4.

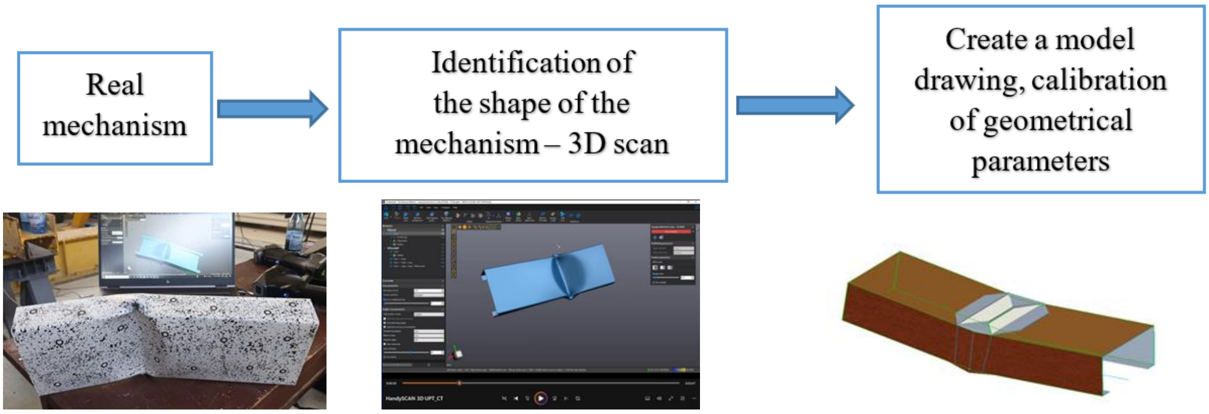

Figure 4: Example of the calibration of the mechanisms model; eccentric compression around minor axis, member 150×60×20, t = 2 mm.

4 LOAD CAPACITY UPPER BOUND ESTIMATION RESULTS

4.1. Plastic mechanisms database

The authors, in accompanying papers [16,19] developed plastic mechanism models and analytical formulations for these mechanisms to characterise the post-ultimate strength of members under investigation subjected to eccentric compression around the minor axis.

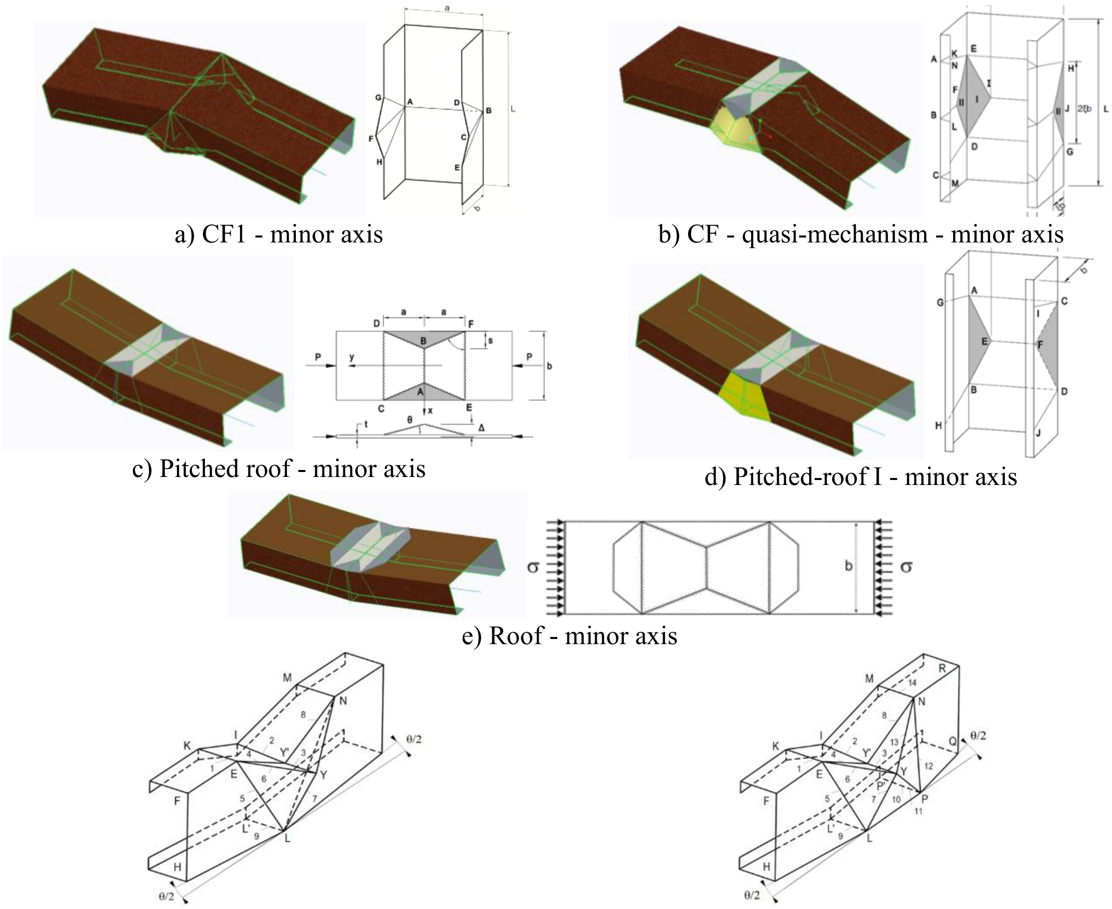

Figure 5: Plastic mechanisms models.

Based on both experimental observations and numerical results from the FE, five plastic mechanism models were confirmed. These mechanisms were fully calibrated, and the post-ultimate mechanism curves derived using YLA, based on their theoretical models, were in good agreement with the FE simulations and the experimental results [17].

Two theoretical models of plastic collapse mechanisms were developed for members subjected to eccentric compression around the major axis [19], also on the basis of experiments and the FE numerical results. The geometry of the real mechanisms obtained in detail using scanning technique indicates the necessity of further calibration of the theoretical models (see Fig. 4). The geometry of the mechanisms is shown in Fig. 5.

4.2. Ultimate strength – comparison of results

Based on the methodology above, the ultimate strength of the members under investigation was determined using the upper bound approach (post-buckling and post-ultimate YL analysis). The upper bound estimation was compared with the experimental and FE results, as well as standard predictions [3]. Exemplary diagrams of the equilibrium paths of the member subjected to eccentric compression around the minor axis are shown in Figure 6. Analogous exemplary diagrams of the equilibrium paths of the member subjected to eccentric compression around the major axis are shown in Figure 7.

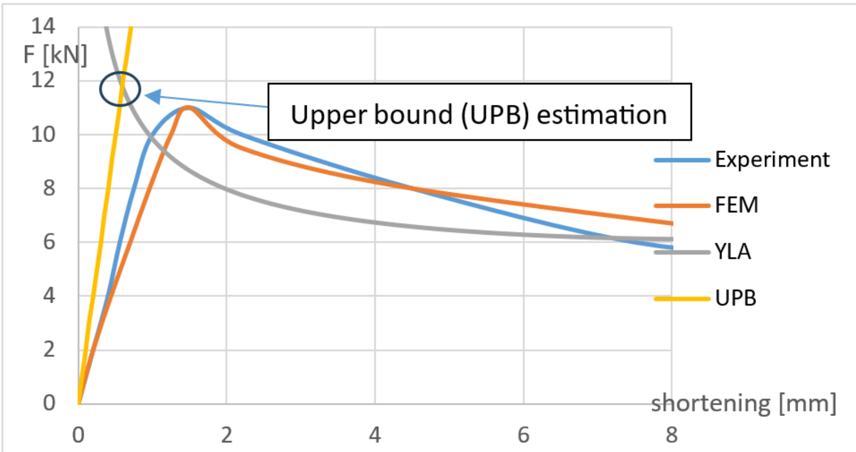

Figure 6: Equilibrium path in the whole range of loading. 150×60×20×1 mm lipped channel, e= - 60 mm (minor axis), YLA – mechanism “Pitched Roof I” (see Figure 5).

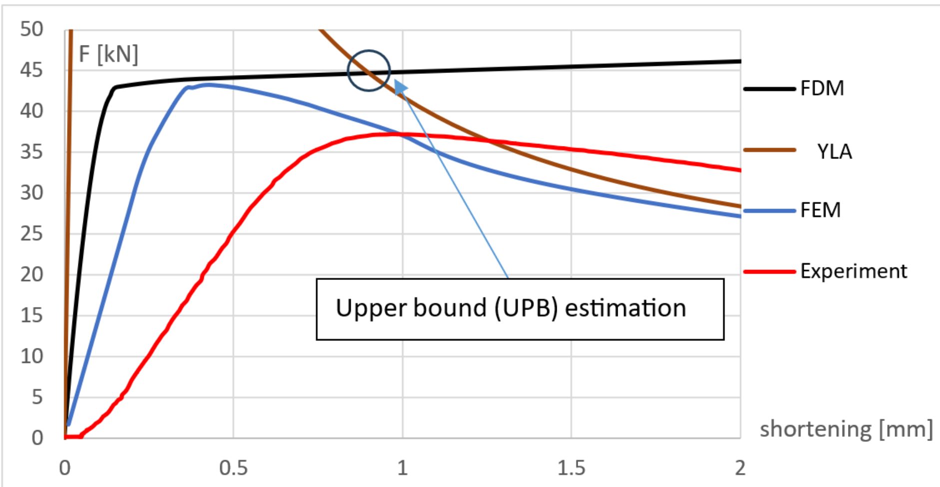

Figure 7: Equilibrium path in the whole range of loading. 150×47×16×1.5 mm lipped channel, e= 60 mm (major axis), YLA – mechanisms “V” (see Figure 5).

The comparative exemplary diagrams of the ultimate strength versus the eccentricity of members subject to eccentricity around the minor axis are shown in Figure 8.

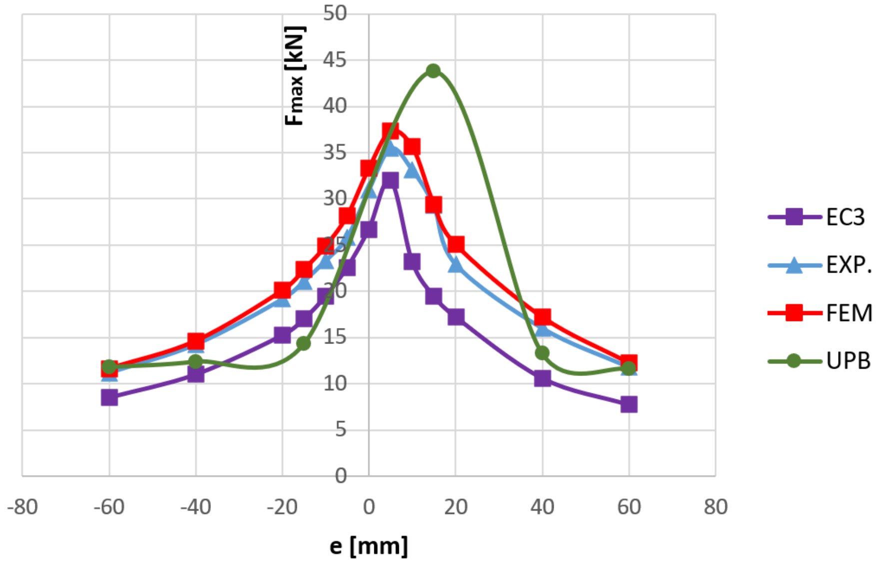

Figure 8: Ultimate strength versus eccentricity around the minor axis for 150×60×20×1 mm lipped channel.

The comparative exemplary diagrams of the ultimate strength for members subject to eccentricity around the major axis are shown in Figure 9.

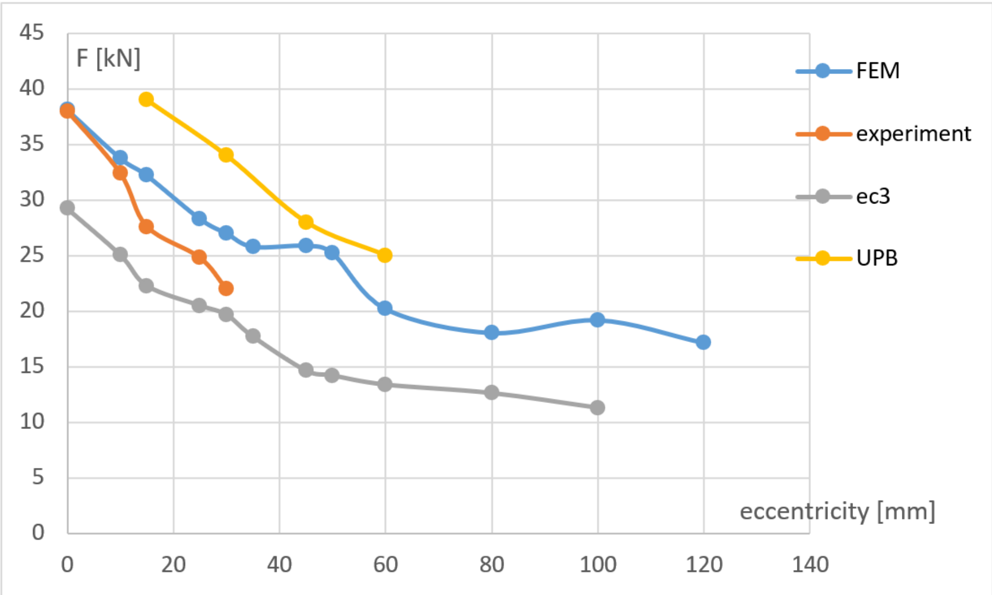

Figure 9: Ultimate strength versus eccentricity for 150×60×20×1 mm lipped channel.

The comparative diagram of the ultimate strength subjected to eccentric compression around the major axis for the entire range of eccentricity and all members tested is shown in Figure 10.

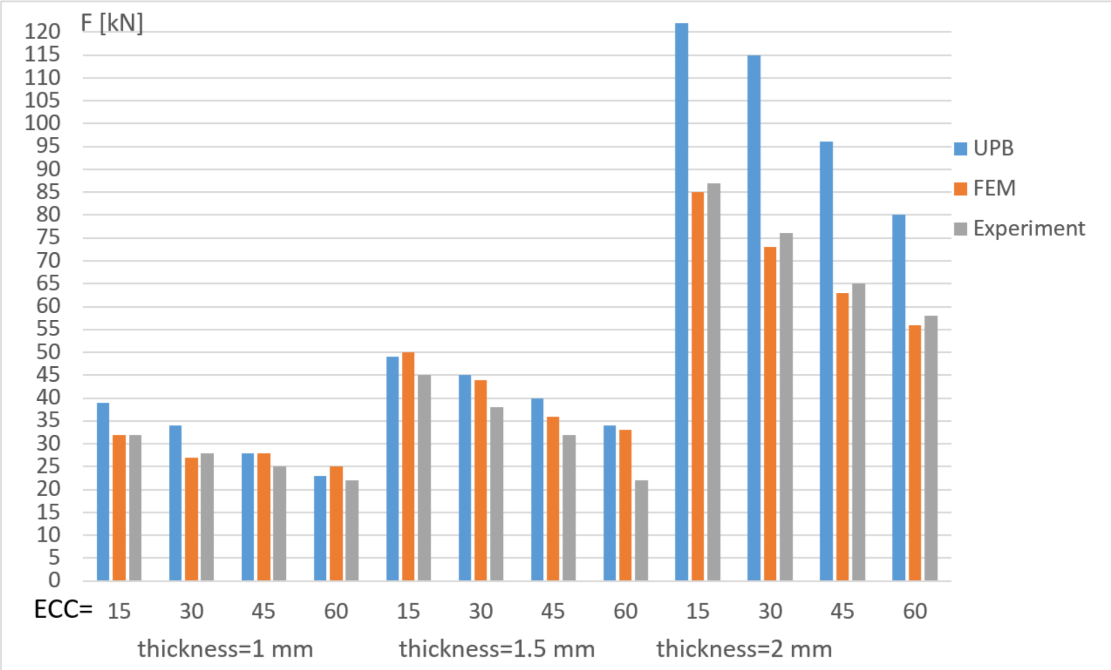

Figure 10: Ultimate strength versus eccentricity – specification for members of different wall thickness.

5 FINAL REMARKS

In the paper, an analytical-numerical study of the application of YLA to the upper bound estimation of ultimate strength (UPB) of short TWCFS lipped channel section members subjected to eccentric compressive load around the minor and major axes was carried out. The results obtained confirm the applicability of the YLA approach accompanied by pre- and post-buckling analysis to the estimation of the load carrying capacity of the members under investigation.

Agreement with FE and experiment with the upper bound prediction for members subjected to eccentric compression on the minor axis is satisfactory. In the case of the major axis, particularly for members of the highest wall thickness (2 mm) upper bound estimation is not entirely convergent with the FE and the experimental results. It indicates the necessity of further calibration of the theoretical plastic mechanism models in that case (as mentioned above).

The 3D laser scanning technique allows to investigate initial imperfections and post-ultimate failure mode, which is necessary to develop theoretical plastic mechanisms models used subsequently in YLA analysis. This technique is particularly useful in the calibration of plastic mechanism models.

Standard predictions of the ultimate strength in the examined case (members under eccentric compression - combined compression and bending) are conservative in comparison with FE simulations and experimental values. Therefore, the UPB ultimate strength prediction method, using the YLA approach presented in the present study, is promising. Further research into an improvement of the method, particularly improving of plastic mechanism theoretical models and creating relatively simple algorithms leading to UPB evaluation, will be continued.

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